wing Becker and Döring [14] one can represent this functional relationship in contributions to the free energy, in which the strain components enter with ascending powers :

$$F(\alpha, e_{ij}) = F_A(\alpha) + F_M(\alpha, e_{ij}) + F_E(e_{ij}) \qquad (1)$$

where F_A, F_M and F_E are the magnetic anisotropy, the magneto-elastic and the elastic energies, respectively. Symmetry arguments dictate the special forms in which these energies can be expressed in the direction cosines α_i and in the strain components.

Taking the cube axes as reference directions one can write for crystals with cubic symmetry :

$$F_A = K_1 s + K_2 p + K_3 s^2 + \cdots \qquad (2)$$

with K_1, K_2, etc. the magnetic anisotropy constants, s equal to $\alpha_1^2 \alpha_2^2 + \alpha_2^2 \alpha_3^2 + \alpha_3^2 \alpha_1^2$, and p equal to $\alpha_1^2 \alpha_2^2 \alpha_3^2$. This expansion, that is commonly used, is less useful for a detailed description of the magnetic anisotropy energy of nickel at low temperature [15]. A development of this energy in the cubic harmonics H_l will be used as an alternative to represent the experimental data for nickel in this temperature region :

$$F_A = \sum_{l=4,6,\ldots} k_l H_l \qquad (3)$$

where the k_l are the corresponding anisotropy constants.

Restricting the expansion of the magneto-elastic energy to the fourth power in the direction cosines α_i, one can express the spontaneous deformation upon rotation of **M** in the magnetostriction constants $h_1, \ldots, h_5$ [16].

Two experimental methods are suited to study the magnetic anisotropy energy in detail. Recent torque [15] as well as magnetization [17] experiments on the magnetic anisotropy energy of nickel report the same features for the constants k_4, k_6 and k_8 at low temperature. An advantage of the magnetization method is the rapid and simple registration of the experimental results. The analysis of the experiments is simpler in torque experiments. The accuracy of ferromagnetic resonance experiments on the magnetic anisotropy energy of nickel and iron is below that of the other methods, given before. The relatively large value of the linewidth, in particular in experiments on bulk material of nickel, makes an accurate determination of the resonance fields impossible. For a detailed study of the higher order contributions to the magnetic anisotropy energy of nickel this method, that requires a non-linear least squares adjustment in evaluating the anisotropy constants, can not be applied [18].

The first two methods can also be used for an investigation of the anisotropy energy under pressure. Pressure experiments by the magnetization process have as an advantage that the experimental situation is not disturbed by the mechanical connection of the high pressure vessel with the pressure generating system [19, 20]. In torque measurements under pressure some specific problems have to be solved. In experiments where the whole torque equipment is placed inside the high pressure vessel the information about the magnetic torque must be brought outside the high pressure vessel by electrical or optical means [21]. Moreover, the effect of pressure on the torsion rigidity of the torsion wire has to be known. Another solution for these problems has been found by using the high pressure tubing itself as a torsion wire in the torque experiments [22]. A limit is set in this case to the sensitivity of the torque measurements, since high pressure tubing is not available in all desired dimensions. In both cases small changes in the maximum torque, due to variations in the pressure, can be observed. Using the second method the effect of pressure on the magnetic anisotropy energy can be followed over a large temperature region.

The application of the strain gauge technique in the magnetostriction problem turns out to be successful. The temperature dependence of the magnetostriction constants and the higher order contributions to the magnetostriction have been studied in this way. The use of this technique in experiments under pressure is not straightforward. The problems of cementing the strain gauges on the sample surface are very serious by a penetration of the gas under the backing material. Besides these problems, that could be solved by using strain gauges with a paper base, one has to measure the gauge factor of the strain gauges as a function of pressure. It was possible to obtain with this technique reproducible results for the effect of pressure on the magnetostriction constants of nickel and iron [22].

III. **Experimental data.** — This discussion will be restricted mainly to the case of nickel where a large spread in the literature data exists.

In the earlier work two constants K_1 and K_2 have been used to describe the experimental results for nickel. The data for K_1 vary at 77 °K gradually from -54 to -84×10^4 erg/cm^3 [23, 24, 25, 26, 27, 15, 17]. Analysing these experimental results we concluded that trivial circumstances could not be responsible for this large spread and that a physical interpretation had to be sought [18]. In order to clear up this question the influence of small amounts of Fe, Co and Cu on the magnetic anisotropy energy of nickel has been investigated [28]. Some of the results with impurity percentages in the order of 0.1 and 1 percent are given in table I. It turns out that the large spread in

TABLE I

Values of the first magnetic anisotropy constant k_4 for nickel and some nickel alloys at 4.2 °K in 10^4 erg/cm^3.

Ni	+ .1 % Cu	+ .14 % Fe	+ .9 % Cu	+ 1.0 % Co
37.3	37.0	35.3	33.9	27.5

the literature results for K_1 of nickel can very well be ascribed to small differences in the purities of the different samples. Literature values for K_2 of nickel not only differ in absolute value but even in sign. Most of these values have been obtained from experiments in the (111) plane, in which plane the anisotropy is determined in principle by K_2 only. Additional torques, partly field dependent, make the torque experiments in this plane very complicated and ask a careful analysis of the experimental data in order to obtain a reliable value for K_2. A Fourier analysis of these additional torques can be helpful in determining

the crystallographic directions in this plane since certain relations between the Fourier coefficients have to be satisfied [15]. A reinterpretation of some of the literature data on the anisotropy in this plane brings the results for K_2 into coincidence over a large temperature region [29]. Values for K_2 from experiments in the (110) plane are in good agreement with these results. The same agreement can be found with the results of Rodbell's ferromagnetic resonance experiments [30] after modification of the resonance condition in the [110] direction in the (111) plane. This implies that there is no experimental evidence that static and dynamic methods yield different values for the anisotropy constants of nickel.

In the past few years it has become clear that a third and even higher anisotropy constants are needed for a description of the experimental results at low temperature [31, 15, 17]. The complex character of the magnetic anisotropy energy at low temperature can be demonstrated by a special plot in which the torque data in the (100) plane are divided by the factor sin $(2\theta \times \cos 2\theta)$, in order to separate the contributions of K_1 to the angle dependence from those of the higher constants, as follows from the expression :

$$L_A/(\sin 2\theta \times \cos 2\theta) = B_1 + B_2 \sin^2 2\theta + B_3 \sin^4 2\theta + \cdots \quad (4)$$

with $B_1 = K_1$, $B_2 = \frac{1}{2} K_3$, etc. and with θ the angle between the direction of **M** and the [001] direction, see figure 1.

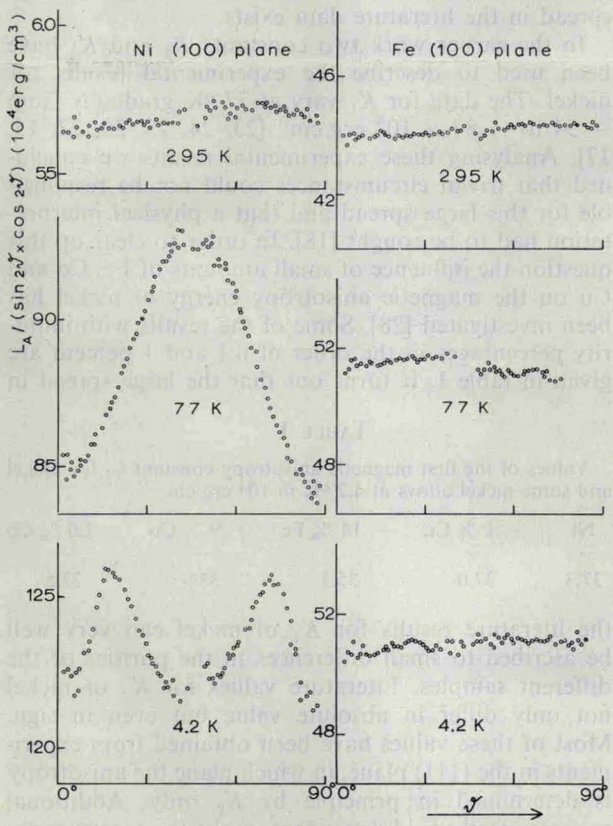

FIG. 1. — Higher order contributions to the torque curves of Ni and Fe in the (100) plane. In order to eliminate contributions from K_1 to the angle dependence, the experimental data have been multiplied with the factor $(\sin 2\theta_i \times \cos 2\theta_i)^{-1}$. (After ref. [15] and [18]).

This plot turns out to be very helpful to show the complicated structure of the anisotropy energy at low temperature. It clearly demonstrates that it is hard to describe the results at 4.2 °K with a small number of the K_i. The anisotropy constants K_1, K_2 and K_3, given in figure 2, have been obtained from

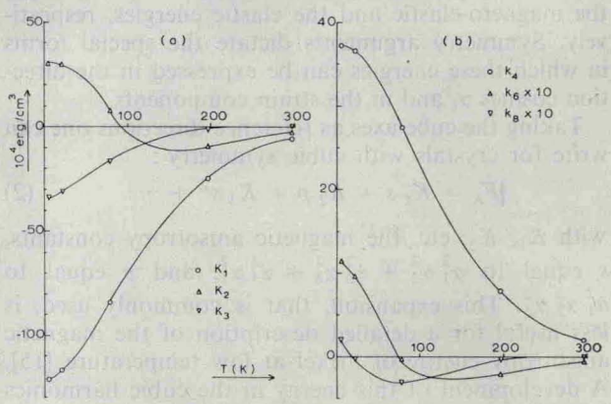

FIG. 2. — Temperature dependence of the magnetic anisotropy constants of nickel.
(*a*) in a description of the energy with eq. (2) ;
(*b*) with eq. (5).
(Data obtained from ref. [15].)

an analysis of the experimental results in the neighbourhood of the [001] direction in the (110) and the (100) plane. A least squares determination of the coefficients in the expansion of the anisotropy energy in cubic harmonics results in a more reliable representation of the experimental results over the full range of orientations. Values for the first three constants in this expansion are also presented in figure 2. It should be noted that these three constants are far from capable to describe the experiments at 4.2 °K within the experimental accuracy. Even a description with six constants does not reveal some specific fine details in the torque curve [15].

The procedure, that for nickel leads to a clear demonstration of higher order contributions to the magnetic torque, results for iron in a flat curve, indicating that only one anisotropy constant has to be used in the (100) plane below room temperature.

In experiments under pressure too the magnetic anisotropy energies of nickel and iron behave differently. The influence of pressure on K_1 of iron is at room temperature nearly the same as at 77 °K. The effect of pressure on the absolute value of K_1 for nickel increases with a factor of about four going from room temperature to 77 °K. The literature data on the pressure effect in the magnetic anisotropy energy, obtained by torque and magnetization methods, agree very well [20, 21, 22].

The information about the magnetostriction constants of iron and nickel is less extensive than that on the magnetic anisotropy constants. Irrespective of a number of details there is uniformity about the main points in the magnetostriction of nickel [33, 34, 35] and iron [5, 32] : the first two constants of nickel and the second constant of iron decrease gradually with increasing temperature ; the first constant of iron shows